NAG C Library Function Document

nag_complex_sparse_eigensystem_init (f12anc)

1 Purpose

nag_complex_sparse_eigensystem_init (f12anc) is a setup function in a suite of functions consisting of nag_complex_sparse_eigensystem_init (f12anc), nag_complex_sparse_eigensystem_iter (f12apc), nag_complex_sparse_eigensystem_sol (f12aqc), nag_complex_sparse_eigensystem_option (f12arc) and nag_complex_sparse_eigensystem_monit (f12asc). It is used to find some of the eigenvalues (and optionally the corresponding eigenvectors) of a standard or generalized eigenvalue problem defined by complex non-symmetric matrices.

The suite of functions is suitable for the solution of large sparse, standard or generalized, non-symmetric complex eigenproblems where only a few eigenvalues from a selected range of the spectrum are required.

2 Specification

```
#include <nag.h>
#include <nagf12.h>
```

```
void nag_complex_sparse_eigensystem_init (Integer n, Integer nev, Integer ncv,
     Integer icomm[], Integer licomm, Complex comm[], Integer lcomm,
     NagError *fail)
```

3 Description

The suite of functions is designed to calculate some of the eigenvalues, λ, (and optionally the corresponding eigenvectors, x) of a standard complex eigenvalue problem $Ax = \lambda x$, or of a generalized complex eigenvalue problem $Ax = \lambda Bx$ of order n, where n is large and the coefficient matrices A and B are sparse, complex and non-symmetric. The suite can also be used to find selected eigenvalues/ eigenvectors of smaller scale dense, complex and non-symmetric problems.

nag_complex_sparse_eigensystem_init (f12anc) is a setup function which must be called before nag_complex_sparse_eigensystem_iter (f12apc), the reverse communication iterative solver, and before nag_complex_sparse_eigensystem_option (f12arc), the options setting function. nag_complex_sparse_eigensystem_sol (f12aqc), is a post-processing function that must be called following a successful final exit from nag_complex_sparse_eigensystem_iter (f12apc), while nag_complex_sparse_eigensystem_monit (f12asc) can be used to return additional monitoring information during the computation.

This setup function initializes the communication arrays, sets (to their default values) all options that can be set by you via the option setting function nag_complex_sparse_eigensystem_option (f12arc), and checks that the lengths of the communication arrays as passed by you are of sufficient length. For details of the options available and how to set them see Section 10.2 of the document for nag_complex_sparse_eigensystem_option (f12arc).

4 References

Lehoucq R B (2001) Implicitly Restarted Arnoldi Methods and Subspace Iteration *SIAM Journal on Matrix Analysis and Applications* **23** 551–562

Lehoucq R B and Scott J A (1996) An evaluation of software for computing eigenvalues of sparse nonsymmetric matrices *Preprint MCS-P547-1195* Argonne National Laboratory

Lehoucq R B and Sorensen D C (1996) Deflation Techniques for an Implicitly Restarted Arnoldi Iteration *SIAM Journal on Matrix Analysis and Applications* **17** 789–821

Lehoucq R B, Sorensen D C and Yang C (1998) *ARPACK Users' Guide: Solution of Large-Scale Eigenvalue Problems with Implicitly Restarted Arnoldi Methods* SIAM, Philidelphia

5 Arguments

1: **n** – Integer *Input*

On entry: the order of the matrix A (and the order of the matrix B for the generalized problem) that defines the eigenvalue problem.

Constraint: $\mathbf{n} > 0$.

2: **nev** – Integer *Input*

On entry: the number of eigenvalues to be computed.

Constraint: $0 < \mathbf{nev} < \mathbf{n} - 1$.

3: **ncv** – Integer *Input*

On entry: the number of Arnoldi basis vectors to use during the computation.

At present there is no *a priori* analysis to guide the selection of **ncv** relative to **nev**. However, it is recommended that $\mathbf{ncv} \geq 2 \times \mathbf{nev} + 1$. If many problems of the same type are to be solved, you should experiment with varying **ncv** while keeping **nev** fixed for a given test problem. This will usually decrease the required number of matrix-vector operations but it also increases the work and storage required to maintain the orthogonal basis vectors. The optimal 'cross-over' with respect to CPU time is problem dependent and must be determined empirically.

Constraint: $\mathbf{nev} + 1 < \mathbf{ncv} \leq \mathbf{n}$.

4: **icomm**[*dim*] – Integer *Communication Array*

Note: the dimension, *dim*, of the array **icomm** must be at least $\max(1, \mathbf{licomm})$.

On exit: contains data to be communicated to the other functions in the suite.

5: **licomm** – Integer *Input*

On entry: the dimension of the array **icomm** as declared in the function from which nag_complex_sparse_eigensystem_init (f12anc) is called.

If $\mathbf{licomm} = -1$, a workspace query is assumed and the function only calculates the required dimensions of **icomm** and **comm**, which it returns in $\mathbf{icomm}(1)$ and $\mathbf{comm}(1)$ respectively.

Constraint: $\mathbf{licomm} \geq 140$ or $\mathbf{licomm} = -1$.

6: **comm**[*dim*] – Complex *Communication Array*

Note: the dimension, *dim*, of the array **comm** must be at least $\max(1, \mathbf{lcomm})$.

On exit: contains data to be communicated to the other functions in the suite.

7: **lcomm** – Integer *Input*

On entry: the dimension of the array **comm** as declared in the function from which nag_complex_sparse_eigensystem_init (f12anc) is called.

If $\mathbf{lcomm} = -1$, a workspace query is assumed and the function only calculates the required dimensions of **icomm** and **comm**, which it returns in $\mathbf{icomm}(1)$ and $\mathbf{comm}(1)$ respectively.

Constraint: $\mathbf{lcomm} \geq 3 \times \mathbf{n} + 3 \times \mathbf{ncv} \times \mathbf{ncv} + 5 \times \mathbf{ncv} + 60$ or $\mathbf{lcomm} = -1$.

8: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.6 of the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument $\langle\mathit{value}\rangle$ had an illegal value.

NE_INT

On entry, **n** ≤ 0: **n** $= \langle\mathit{value}\rangle$.

On entry, **nev** ≤ 0: **nev** $= \langle\mathit{value}\rangle$.

NE_INT_2

The length of the integer array **icomm** is too small **licomm** $= \langle\mathit{value}\rangle$, but must be at least $\langle\mathit{value}\rangle$.

NE_INT_3

On entry, **lcomm** $< 3 \times$ **n** $+ 3 \times$ **ncv** $\times$ **ncv** $+ 5 \times$ **ncv** $+ 60$ where: **lcomm** $= \langle\mathit{value}\rangle$, **n** $= \langle\mathit{value}\rangle$, **ncv** $= \langle\mathit{value}\rangle$.

On entry, **ncv** $\leq$ **nev** $+ 1$ or **ncv** $>$ **n** where: **ncv** $= \langle\mathit{value}\rangle$, **nev** $= \langle\mathit{value}\rangle$, **n** $= \langle\mathit{value}\rangle$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

Not applicable.

8 Further Comments

None.

9 Example

The example solves $Ax = \lambda x$ in regular mode, where A is obtained from the standard central difference discretization of the convection-diffusion operator $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \rho\frac{\partial u}{\partial x}$ on the unit square, with zero Dirichlet boundary conditions. The eigenvalues of largest magnitude are found.

9.1 Program Text

```
/* nag_complex_sparse_eigensystem_init (f12anc) Example Program.
 *
 * Copyright 2005 Numerical Algorithms Group.
 *
 * Mark 8, 2005.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <nag_string.h>
#include <stdio.h>
#include <naga02.h>
#include <nagf12.h>
#include <nagf16.h>
static void av(Integer, Complex *, Complex *);
static void tv(Integer, Complex *, Complex *);


int main(void)
{
  /* Constants */
  Integer licomm=140, imon=0;
  /* Scalars */
  Complex sigma;
  double estnrm;
  Integer exit_status, i, irevcm, lcomm, n, nconv, ncv;
  Integer nev, niter, nshift, nx;
  /* Nag types */
  NagError fail;

  /* Arrays */
  Complex *comm=0, *eigest=0, *eigv=0, *resid=0, *v=0;
  Integer *icomm=0;
  /* Ponters */
  Complex *mx=0, *x=0, *y=0;

  exit_status = 0;
  INIT_FAIL(fail);

  Vprintf("nag_complex_sparse_eigensystem_init (f12anc) Example Program "
          "Results\n");
  /* Skip heading in data file */
  Vscanf("%*[^\n] ");

  Vscanf("%ld%ld%ld%*[^\n] ", &nx, &nev, &ncv);
  n = nx * nx;
  lcomm = 3*n + 3*ncv*ncv + 5*ncv + 60;
  /* Allocate memory */
  if ( !(comm = NAG_ALLOC(lcomm, Complex)) ||
       !(eigv = NAG_ALLOC(ncv, Complex)) ||
       !(eigest = NAG_ALLOC(ncv, Complex)) ||
       !(resid = NAG_ALLOC(n, Complex)) ||
       !(v = NAG_ALLOC(n * ncv, Complex)) ||
       !(icomm = NAG_ALLOC(licomm, Integer)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }
  /* Initialise communication arrays for problem using
     nag_complex_sparse_eigensystem_init (f12anc). */
  nag_complex_sparse_eigensystem_init(n, nev, ncv, icomm, licomm,
                                      comm, lcomm, &fail);
  irevcm = 0;
 REVCOMLOOP:
  /* repeated calls to reverse communication routine
     nag_complex_sparse_eigensystem_iter (f12apc). */
  nag_complex_sparse_eigensystem_iter(&irevcm, resid, v, &x, &y, &mx,
                                      &nshift, comm, icomm, &fail);
  if (irevcm != 5 && irevcm != 0)
    {
      if (irevcm == -1 || irevcm == 1)
        {
          /* Perform matrix vector multiplication y <--- Op*x */
          av(nx, x, y);
        }
      else if (irevcm == 4 && imon == 1)
        {
          /* If imon=1, get monitoring information using
             nag_complex_sparse_eigensystem_monit (f12asc). */
          nag_complex_sparse_eigensystem_monit(&niter, &nconv, eigv,
                                               eigest, icomm, comm);
          /* Compute 2-norm of Ritz estimates using
             nag_zge_norm (f16uac). */
          nag_zge_norm(Nag_ColMajor, Nag_FrobeniusNorm, nev, 1, eigest,
                       nev, &estnrm, &fail);
          Vprintf("Iteration %3ld, ", niter);
          Vprintf(" No. converged = %3ld,", nconv);
          Vprintf(" norm of estimates = %16.8e\n", estnrm);
        }
      goto REVCOMLOOP;
    }
  if (fail.code == NE_NOERROR)
    {
      /* Post-Process using nag_complex_sparse_eigensystem_sol
         (f12aqc) to compute eigenvalues/vectors. */
      nag_complex_sparse_eigensystem_sol(&nconv, eigv, v, sigma,
                                          resid, v, comm, icomm, &fail);

      Vprintf("\n The %ld Ritz values", nconv);
      Vprintf(" of largest magnitude are:\n\n");
      for (i = 0; i <= nconv-1; ++i)
        {
          Vprintf("%8ld%5s(%12.4f, %12.4f)\n", i+1,  "",
                  eigv[i].re, eigv[i].im);
        }
    }
  else
    {
      Vprintf(" Error from nag_complex_sparse_eigensystem_iter (f12apc)."
              "\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  if (comm) NAG_FREE(comm);
  if (eigv) NAG_FREE(eigv);
  if (eigest) NAG_FREE(eigest);
  if (resid) NAG_FREE(resid);
  if (v) NAG_FREE(v);
  if (icomm) NAG_FREE(icomm);
  return exit_status;
}

static void av(Integer nx, Complex *x, Complex *y)
{
  /* Scalars */
  double hr;
  Integer i, j, lo;
  /* Function Body */

  /* Allocate memory */
  hr = (double) -(nx + 1) * (nx + 1);
  tv(nx, x, y);
  for (j = 0; j <= nx - 1; ++j)
    {
      y[j].re = y[j].re + hr*x[nx+j].re;
      y[j].im = y[j].im + hr*x[nx+j].im;
    }
  for (j = 2; j <= nx - 1; ++j)
    {
      lo = (j - 1) * nx;
      tv(nx, &x[lo], &y[lo]);
      for (i = 0; i <= nx - 1; ++i)
        {
          y[lo+i].re = y[lo+i].re + hr*(x[lo-nx+i].re+x[lo+nx+i].re);
          y[lo+i].im = y[lo+i].im + hr*(x[lo-nx+i].im+x[lo+nx+i].im);
        }
    }
  lo = (nx - 1) * nx;
  tv(nx, &x[lo], &y[lo]);
  for (j = 0; j <= nx - 1; ++j)
    {
      y[lo+j].re = y[lo+j].re + hr*x[lo-nx+j].re;
      y[lo+j].im = y[lo+j].im + hr*x[lo-nx+j].im;
    }
} /* av */


static void tv(Integer nx, Complex *x, Complex *y)
{
  /* Compute the matrix vector multiplication y<---T*x where T is a */
  /* nx by nx tridiagonal matrix. */

  /* Scalars */
  Complex dd, dl, du, h2, h, rho, z1, z2, z3;
  Integer j;

  /* Function Body */
  /* Assign to Complex type using nag_complex (a02bac) */
  h  = nag_complex((double) (nx + 1),0.);
  /* Compute Complex multiply using nag_complex_multiply (a02ccc). */
  h2 = nag_complex_multiply(h, h);
  dd = nag_complex_multiply(nag_complex(4.0,0.0), h2);
  z1 = nag_complex_multiply(nag_complex(-1.0, 0.0), h2);
  /* Assign to Complex type using nag_complex (a02bac) */
  rho = nag_complex(1.0e2, 0.0);
  z2 = nag_complex_multiply(rho, h);
  z3 = nag_complex_multiply(nag_complex(5.0e-1, 0.0),z2);
  /* Compute Complex subtraction using nag_complex_subtract
     (a02cbc). */
  dl = nag_complex_subtract(z1, z3);
  /* Compute Complex addition using nag_complex_add (a02cac). */
  du = nag_complex_add(z1, z3);

  /* Compute Complex multiply using nag_complex_multiply (a02ccc). */
  z1 = nag_complex_multiply(dd, x[0]);
  z2 = nag_complex_multiply(du, x[1]);
  /* Compute Complex addition using nag_complex_add (a02cac). */
  y[0] = nag_complex_add(z1,z2);
  for (j = 1; j <= nx - 2; ++j)
    {
      /* Compute Complex multiply using nag_complex_multiply
         (a02ccc). */
      z1 = nag_complex_multiply(dl, x[j-1]);
      z2 = nag_complex_multiply(dd, x[j]);
      z3 = nag_complex_multiply(du, x[j+1]);
      /* Compute Complex addition using nag_complex_add (a02cac). */
      y[j] = nag_complex_add(z1, z2);
      y[j] = nag_complex_add(y[j], z3);
    }
  /* Compute Complex multiply using nag_complex_multiply (a02ccc). */
  z1 = nag_complex_multiply(dl, x[nx-2]);
  z2 = nag_complex_multiply(dd, x[nx-1]);
  /* Compute Complex addition using nag_complex_add (a02cac). */
  y[nx-1] = nag_complex_add(z1, z2);
  return;
} /* tv */
```

9.2 Program Data

```
nag_complex_sparse_eigensystem_init (f12anc) Example Program Data
 10 4 20 : Vaues for nx, nev and ncv
```

9.3 Program Results

```
nag_complex_sparse_eigensystem_init (f12anc) Example Program Results

 The 4 Ritz values of largest magnitude are:

       1     (    716.1973,   -1029.5838)
       2     (    687.5834,   -1029.5838)
       3     (    716.1973,    1029.5838)
       4     (    687.5834,    1029.5838)
```